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Establishing a No-Lose Theorem for NMSSM Higgs Boson Discovery at the LHC

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Abstract

We scan the parameter space of the NMSSM for the observability of at least one Higgs boson at the LHC with 300 fb^{-1} integrated luminosity, taking the present LEP2 constraints into account. We find that if WW-fusion detection modes for a light Higgs boson are not taken into account, then there are still significant regions in the parameter space of the NMSSM where no Higgs boson can be observed at the 5σ level, despite the recent improvements in ATLAS and CMS procedures and techniques and even if we combine all non-fusion discovery channels. However, if the WW-fusion detection modes are included using the current theoretical study estimates, then we find that for all points at least one of the NMSSM Higgs bosons will be detected. In a few cases, a 5σ signal cannot be obtained in any one channel but will require combining signals of lower statistical significance from several modes. If the estimated 300 fb^{-1} significances for ATLAS and CMS are combined, one can achieve 5σ signals after combining just the non-WW-fusion channels signals. We present the parameters of several particularly difficult points, and discuss the complementary roles played by different modes. We conclude that the LHC will discover at least one NMSSM Higgs boson unless there are large branching ratios for decays to SUSY particles and/or to other Higgs bosons.

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1 Introduction

Supersymmetric extensions of the standard model generally predict relatively light Higgs bosons. One of the most important tasks of the LHC is the search for Higgs bosons [1, 2]. An important milestone in understanding the potential of the LHC was the demonstration that at least one Higgs boson of the minimal supersymmetric standard model (MSSM) would be detectable at the $\geq 5\sigma$ level throughout all of the MSSM parameter space so long as top squark masses do not exceed 1.5 to 2 TeV and so long as large branching fractions to decay channels containing supersymmetric particles are not substantial.

In the present paper, we study, subject to these same and a few other simplifying restrictions, the detectability of Higgs bosons in the next-to-minimal supersymmetric standard model (NMSSM). In the NMSSM, one Higgs singlet superfield, $\widehat{S}$, is added to the MSSM in order to render unnecessary the bilinear superpotential term $\mu \widehat{H}_1 \widehat{H}_2$ by replacing it with $\lambda \widehat{S} \widehat{H}_1 \widehat{H}_2$, where the vacuum expectation value of the scalar component of $\widehat{S}$, $\langle S \rangle$, results in an effective bilinear Higgs mixing with $\mu = \lambda \langle S \rangle$. The detectability of the NMSSM Higgs bosons was first considered in a contribution to Snowmass 96 [3]. The result, using the experimentally established modes and sensitivities available at the time, was that substantial regions in the parameter space of the NMSSM were found where none of the Higgs bosons would have been observable either at LEP2 or at the LHC even with an integrated luminosity of 600 fb^{-1} (two detectors with $L = 300$ fb^{-1} each).

Since then, progress has been made both on the theoretical and the experimental sides. On the theoretical side, the dominant two-loop corrections to the effective potential of the model have been computed [4, 5]. These lead to a modest decrease in the mass of the lightest Higgs scalar, holding fixed the stop sector parameters. Inclusion of the two-loop corrections thus increases somewhat the part of the NMSSM parameter space excluded by LEP2 (and accessible at the Tevatron) [5], but is of less relevance for the LHC. On the experimental side the expected statistical significances have been improved since 1996 [1, 2]. Most notably, associated $t\bar{t}h$ production with $h \to b\bar{b}$ (originally discussed in [6]), which in the SM context is particularly sensitive to $m_h \lesssim 120$ GeV, has been added by ATLAS and CMS to the list of Higgs boson detection modes [1, 2]. Analysis of this mode was recently extended [7] to $m_h = 140$ GeV, which, though not relevant in the SM case due to the decline in the $b\bar{b}$ branching ratio as the WW^* mode increases, is highly relevant for points in our searches for which the WW^* mode is suppressed in comparison to the SM prediction. In addition, techniques have been proposed [8] for isolating signals for WW fusion to a light Higgs boson which decays to $\tau\overline{\tau}$ or $WW^{(*)}$.

It turns out that adding in just the $t\bar{t}h$ process renders the no-Higgs-discovery parameter choices described and plotted in [3], including the "black point" described in detail there, visible [9]. In the present paper, we search for any remaining parameter choices for which no Higgs boson would produce a $\geq 5\sigma$ signal. In this search, we perform a scan over nearly all of the parameter space of the model, the only parameter choices not included being those for which there is sensitivity to highly model-dependent decays. The outcome is that, for an integrated luminosity of 300 fb^{-1} at the LHC, there are still regions in the parameter space with $< 5\sigma$ expected statistical significance (computed as $N_{SD} = S/\sqrt{B}$ for a given mode) for all Higgs detection modes so far studied in detail by ATLAS and CMS, *i.e.* including the $t\bar{t}h \to t\bar{t}b\bar{b}$ mode but not the WW-fusion modes. On the other hand, the expected statistical significance for at least one of these detection modes is always above 2.8σ at 300 fb^{-1}, and the statistical significance obtained by combining (using the naive Gaussian procedure) all the non-WW-fusion modes is at least 4.4. However, we find that all such cases are quite observable (at $\geq 5.8\sigma$) in one of the WW-fusion modes (using theoretically estimated statistical significances for these modes). Further, we were able to find only one very small region in parameter space for which the best non-WW-fusion mode and the best WW-fusion mode are both predicted to yield statistical significance below 5σ. But, at 4.8σ each, they are only just below 5σ. For all points in the scan of parameter space, statistical significances obtained by combining all modes, including WW-fusion modes, are always $\gtrsim 7\sigma$. Thus NMSSM Higgs discovery by just one detector with $L = 300$ fb^{-1} is essentially guaranteed. By combining results from both the ATLAS and CMS detectors, by further refining the analysis techniques, and/or by accumulating more integrated luminosity it seems clear that there is a "no-lose" theorem for NMSSM Higgs boson discovery at the LHC analogous to that established for the MSSM.

In order to clarify the nature of the most difficult points, we present, in sect. 4, examples of particularly difficult bench mark points for the Higgs sector of the NMSSM. Apart from the "bare" parameters of the model, we give the masses and couplings of all Higgs scalars, their production rates and branching ratios to various channels (relative to the SM Higgs) and details of the statistical significances predicted for each Higgs boson in each channel. The latter will allow an assessment of exactly what level of improvement in statistical significance will be required in the various different detection modes in order to render marginal modes visible. Of course, our estimates of the expected statistical significances are often somewhat crude (e.g. their dependence on the accumulated integrated luminosity). We believe that our procedures always err in the conservative direction, leading to statistical significances that might be a bit small. Thus, the LHC procedures for isolating Higgs boson signals could provide even more robust signals for NMSSM Higgs boson detection than we estimate here.

The detection modes, which serve for the searches for standard model or MSSM Higgs

bosons, include (using the notation h, a for CP-even, CP-odd Higgs bosons, respectively):
1) $gg \to h \to \gamma\gamma$;
2) associated Wh or $t\bar{t}h$ production with $\gamma\gamma\ell^{\pm}$ in the final state;
3) associated $t\bar{t}h$ production with $h \to b\bar{b}$;
4) $gg \to h/a$ or associated $b\bar{b}h/a$ production with $h/a \to \tau\overline{\tau}$;
5) $gg \to h \to ZZ^{(*)} \to 4$ leptons;
6) $gg \to h \to WW^{(*)} \to l^+l^-\nu\overline{\nu}$;
7) LEP2 $e^+e^- \to Z^* \to Zh$;
8) $WW \to h \to \tau\overline{\tau}$;
9) $WW \to h \to WW^{(*)}$,
where 8) and 9) are those analyzed at the theoretical level in [8] and included in the NMSSM analysis for the first time in this paper. The above detection modes do not employ the possibly important decay channels i) $a \to Zh$, ii) $h \to aa$, iii) $h \to hh$, iv) $h, a \to t\bar{t}$ and v) $t \to h^{\pm} + b$. The branching ratios for the decays i)-iii) are highly model-dependent, while the existing analyzes of the $t\bar{t}$ final state signatures are not very detailed. Further, when kinematically allowed, the $t \to h^{\pm} + b$ signal would be easily observed according to existing analyzes. Thus, we restrict our scan over NMSSM parameter space to those parameters for which none of these decays are present.

The Higgs sector of the NMSSM consists of 3 scalars, denoted h_1, h_2, h_3 with $m_{h_1} < m_{h_2} < m_{h_3}$, 2 pseudo-scalars, denoted a_1, a_2 with $m_{a_1} < m_{a_2}$, and a charged Higgs pair, denoted $h^{\pm}$. Mixing of the neutral doublet fields with the gauge singlet fields in the scalar and in the pseudo-scalar sector can be strong. The scalar mixing can lead to a simultaneous suppression of the couplings of all the h_i to gauge bosons, and hence to a suppression of many of the detection modes above. (Of course, the a_i have no tree-level couplings to gauge boson pairs and the one-loop couplings are too small to yield useful event rates.) The couplings of the Higgs bosons to t- or b-quarks can be amplified, reduced or even change sign with respect to the standard model couplings. Hence negative interferences can occur among the (loop-) diagrams contributing to $gg \to h_i$ and $h_i \to \gamma\gamma$, leading again to suppressions of the above detection modes. A complete simultaneous annulation of all detection modes is not possible, but simultaneous reduction of all detection modes is possible and it is for such parameter choices that NMSSM Higgs boson discovery is most difficult.

In addition to restricting our search for "black spots" to parts of parameter space for which the decay modes i) -v) are forbidden, we take the constraints of LEP2 [via the mode 7)] on Higgs scalars with masses below 114 GeV into account, and only accept points for which 5σ discovery at LEP2 would not have been possible. In the next section 2, we define the class of models we are going to consider, and the way we perform the scan over the corresponding parameter space. In section 3 we describe our computations of the expected statistical significances of the detection modes 1) – 6) above. In section 4, we present six particularly difficult bench mark points (in table 1) and details regarding their statistical significances in channels 1)-9) in table 2, with a summary of overall statistical significances in table 3. Using these tables, we give a discussion of the properties of these points.

2 NMSSM Parameters and Scanning Procedure

In this paper, we consider the simplest version of the NMSSM [10, 11, 12], where the term $\mu \widehat{H}_1 \widehat{H}_2$ in the superpotential of the MSSM is replaced by (we use the notation $\widehat{A}$ for the superfield and A for its scalar component field)

$$\lambda \widehat{H}_1 \widehat{H}_2 \widehat{S} \ + \ \frac{\kappa}{3} \widehat{S}^3 \quad , \tag{2.1}$$

so that the superpotential is scale invariant. We make no assumption on "universal" soft terms. Hence, the five soft supersymmetry breaking terms

$$m_{H_1}^2 H_1^2 \ + \ m_{H_2}^2 H_2^2 \ + \ m_S^2 S^2 \ + \ \lambda A_\lambda H_1 H_2 S \ + \ \frac{\kappa}{3} A_\kappa S^3 \tag{2.2}$$

are considered as independent. The masses and/or couplings of sparticles are assumed to be such that their contributions to the loop diagrams inducing Higgs production by gluon fusion and Higgs decay into $\gamma\gamma$ are negligible. For the soft stop mass terms, which are an important input to the radiative corrections to the Higgs sector, we assume 1 TeV. (For lower soft stop masses, the Higgs bosons are lighter and discovery would generally be easier. For higher soft stop masses, the Higgs boson masses are larger and discovery tends to become more difficult.) The absence of Landau singularities for λ and κ below the GUT scale ($\sim 2 \times 10^{16}$ GeV) imposes upper bounds on these couplings at the weak scale, which depend on the value of h_t and hence of $\tan\beta$ [10, 11] (we used $m_{top}^{pole} = 175$ GeV). We assume that the Higgs sector is CP conserving.

The independent parameters of the Higgs sector of the model are thus

$$\lambda, \kappa, m_{H_1}^2, m_{H_2}^2, m_S^2, A_\lambda, A_\kappa \ . \tag{2.3}$$

In the stop sector, which appears in the radiative corrections to the Higgs potential, we chose the soft masses $m_Q = m_T \equiv M_{susy} = 1$ TeV, and varied the stop mixing parameter

$$X_t \equiv 2 \, \frac{A_t^2}{M_{susy}^2 + m_t^2} \left(1 - \frac{A_t^2}{12(M_{susy}^2 + m_t^2)} \right) \ . \tag{2.4}$$

As in the MSSM, the value $X_t = \sqrt{6}$ – so called maximal mixing – maximizes the radiative corrections to the Higgs masses, and we found that it leads to the most challenging points in the parameter space of the NMSSM.

For purposes of scanning and analysis, it is more convenient to eliminate $m_{H_1}^2$, $m_{H_2}^2$ and m_S^2 in favour of M_Z, $\tan\beta$ and $\mu_{\rm eff} = \lambda\langle S\rangle$ through the three minimization equations of the Higgs potential (including the dominant 1- and 2-loop corrections [5]) and to scan over the six independent parameters

$$\lambda, \kappa, \tan\beta, \mu_{\rm eff}, A_\lambda, A_\kappa \ . \tag{2.5}$$

We adopt the convention $\lambda, \kappa > 0$, in which $\tan\beta$ can have either sign. For each point in the parameter space, we then diagonalize the scalar and pseudo-scalar mass matrices and compute the scalar, pseudo-scalar and charged Higgs masses and couplings. We demand that the running Yukawa couplings are free of the above-mentioned Landau singularities and that the Higgs scalars satisfy the LEP2 constraints (taken from [13], fig. 5) on m_{h_i} vs. their couplings R_i to the Z boson. In order to render the above-mentioned processes i) to v) kinematically impossible, we require the following inequalities among the masses: $m_{h_3}/2 < m_{a_i} < M_Z + m_{h_1}$, $m_{h_3} < 2m_{h_1}$ and $m_{h^\pm} > 155$ GeV. In addition we require $|\mu_{\rm eff}| > 100$ GeV; otherwise a light chargino would have been detected at LEPII. (The precise lower bound on $|\mu_{\rm eff}|$ depends somewhat on $\tan\beta$ and the precise experimental lower bound on the chargino mass; however, our subsequent results do not depend on the precise choice of the lower bound on $|\mu_{\rm eff}|$.) We further note that for the most challenging parameter space points that we shall shortly discuss, $|\mu_{\rm eff}| > 100$ GeV is already sufficient to guarantee that the NMSSM Higgs bosons cannot decay to chargino pairs so long as the SU(2) soft-SUSY-breaking parameter M_2 is also large. In fact, in order to avoid significant corrections to $\gamma\gamma h_i$ and $\gamma\gamma a_i$ couplings coming from chargino loops it is easiest to take $M_2 \gg \mu_{\rm eff}$ (or vice versa). This is because the $\gamma\widetilde{\chi}_i^+\widetilde{\chi}_i^-$ coupling is suppressed if the $\widetilde{\chi}_i^+$ is either pure higgsino or pure gaugino. Since the parts of parameter space that are challenging with regard to Higgs detection typically have $|\mu| \sim 100-200$ GeV, the validity of our assumptions requires that M_2 be large and that the chargino be essentially pure higgsino.

Using a very rough sampling, we determined, as expected from previous work, that it is only for moderate values of $\tan\beta$ that $< 5\sigma$ signals might possibly occur. We then performed a more detailed scan focusing on the region $-10 < \tan\beta < 10$ using a large number of λ, κ, $\mu_{\rm eff}$, A_λ and A_κ values. The maximum values of λ and κ are determined by requiring the absence of Landau singularities; for intermediate $\tan\beta$, $\lambda_{\rm max} \sim 0.69$, while $\kappa_{\rm max} \sim 0.62$ for small λ. In total, we sampled some 2.2×10^8 points in parameter space. From this 2nd more detailed sampling, we determined the most difficult parameter space regions and further refined our scan to the following:

- $3.5 < |\tan\beta| < 8$ (both signs) in steps of 0.5;
- $0.001 < \lambda < \min[0.3, \lambda_{\rm max}]$, using 30 points;
- $0.001 < \kappa < \min[0.35, \kappa_{\rm max}]$, using 30 points;
- 100 GeV $< |\mu_{\rm eff}| <$ 600 GeV (both signs), in steps of 10 GeV;
- $0 < |A_\lambda| <$ 160 GeV, with A_λ opposite in sign to $\mu_{\rm eff}$, using steps of 10 GeV;
- 30 GeV $< |A_\kappa| <$ 170 GeV, with A_κ opposite in sign to $\mu_{\rm eff}$, using steps of 10 GeV.

For those points sampled in this final scan which satisfy all the constraints detailed earlier, we compute the expected statistical significances for the processes 1) to 9) listed in section 1 as described in the next section. As a rough guide, from the 4.7×10^8 points detailed in the above list, we find about 4×10^4 that pass all constraints and have $N_{SD} < 5$ (for $L = 300$ fb^{-1}) in each of the individual discovery modes 1) – 7). However, only a very tiny subset of these points (actually a point found after shifting the parameters slightly off the very worst point found among these) is such that the LHC modes 8) – 9) also have $N_{SD} < 5$. We shall tabulate details of this point and a number of representative points taken from the final set of 40,000 points.

3 Expected Statistical Significances

From the known couplings of the NMSSM Higgs scalars to gauge bosons and fermions it is straightforward to compute their production rates in gluon-gluon fusion and various associated production processes, as well as their partial widths into $\gamma\gamma$, gauge bosons and fermions, either relative to a standard model Higgs scalar or relative to the MSSM H and/or A. This allows us to apply "NMSSM corrections" to the processes 1) – 9) above.

These NMSSM corrections are computed in terms of the following ratios. For the scalar Higgs bosons, c_V is the ratio of the coupling of the h_i to vector bosons as compared to that of a SM Higgs boson (the coupling ratios for h_iZZ and h_iWW are the same), and c_t, c_b are the corresponding ratios of the couplings to top and bottom quarks. Since we employ tree-level results for Yukawa couplings, one has $c_\tau = c_b$. Note that we always have $|c_V| < 1$, but c_t and c_b can be larger, smaller or even differ in sign with respect to the standard model. For the CP-odd Higgs bosons, c_V is not relevant since there is no tree-level coupling of the a_i to the VV states; c_t and c_b are defined as the ratio of the $i\gamma_5$ couplings for $t\bar{t}$ and $b\bar{b}$, respectively, relative to SM-like strength.

The expected statistical significances for the processes 1) and 6) are computed beginning with results for the SM Higgs boson taken from ref [14], fig.1 ("Expected Observability of Standard Model Higgs in CMS with 100 fb^{-1}"). The application of the NMSSM corrections using c_V, c_t and c_b [which determine $\Gamma(gg \to h_i)$, $BR(h_i \to \gamma\gamma)$ and $BR(h_i \to WW^*)$] is straightforward in these two cases. †

The expected statistical significances for process 2) are taken from the same figure. In ref. [16] one finds that Wh_i and $t\bar{t}h_i$ production contribute with roughly equal weight to the SM signal. This allows us to decompose the expected significance into the corresponding production processes, apply the NMSSM corrections, and then recombine the production processes.

The expected Standard Model Higgs statistical significances for process 3) are taken from table 19-8 in ref. [1], with the extension to Higgs masses above 120 GeV as provided in [7], using a numerical interpolation for Higgs masses below 140 GeV. For the standard model process 5) we again use ref. [1], tables 19-18 and 19-21. In both cases, the application of the NMSSM corrections is straightforward.

The estimation of the statistical significances for the process 4) in the NMSSM requires the most discussion. Figure 19-62 of ref. [1] and fig. 8 of ref. [16] give the 5σ contours in the $\tan\beta$ - m_A plane of the MSSM. The critical issue is how much of these 5σ signals derive from $gg \to H + gg \to A$ production and how much from associated $b\bar{b}H + b\bar{b}A$ production, and how each of the gg fusion and $b\bar{b}$ associated production processes are divided up between H and A. For the former, we turn to table 19-35 of ref. [1]. There, we see that it is for cuts designed to single out the associated production processes that large statistical significance can be achieved and that such cuts provide 90% of the net statistical significance of $N_{SD} = 8.9$ (3.9 for gg fusion cuts combined in quadrature with 8.0 for $b\bar{b}H+b\bar{b}A$ associated production cuts) for $m_A = 150$ GeV and $L = 30$ fb^{-1}. (For the associated production cuts, the table shows that the contribution of the gg fusion processes to the signal is very small.) The percentage of N_{SD} deriving from gg-fusion cuts is even smaller at high m_A. Since we are mainly interested in $m_H,\ m_A \in [100\text{ GeV}, 200\text{ GeV}]$, we will assume that 90% of the statistical significance along the contours of fig. 19-62 comes from the associated production cut analysis; this will give us a slightly conservative estimate of the associated production N_{SD} values at still higher m_A. With this choice,

† In our computations, we neglect the contribution to the $h_i\gamma\gamma$ coupling coming from the charged Higgs loop. Despite the relatively small masses of the h^+ for our most problematical points, $m_{h^+} \in [155\text{ GeV}, 160\text{ GeV}]$, the charged Higgs loop decouples [15], especially for the small values of λ and λA_λ characteristic of difficult points for which the actual $h^+h^-h_i$ coupling is only of order a few times $gm_W/(4\sqrt{2})$ [10]. Its contribution would typically only be of order a few percent even though our difficult points have smaller $\gamma\gamma h_i$ coupling than a SM-like Higgs boson by virtue of suppressed h_iWW coupling and/or cancellation between the top and W loop contributions.

the 5σ contour at $L = 100\ \text{fb}^{-1}$ from fig. 19-62 of ref. [1] corresponds to a 4.5σ contour for associated $b\overline{b}H + b\overline{b}A$ production alone. Since the values of $\tan\beta$ along this contour are large, we can separate the H and A signals from one another by using the following properties of the MSSM within which fig. 19-62 of ref. [1] was generated: (a) $BR(H \to \tau\overline{\tau}) \sim BR(A \to \tau\overline{\tau}) \sim 0.09$; (b) the $b\overline{b}A$ and $b\overline{b}H$ couplings are very nearly equal and scale as $\tan\beta$; and (c) $m_A \sim m_H$ within the $\tau\overline{\tau}$ mass resolution. As a result, the net signal rate along this contour is approximately twice that for $b\overline{b}A$ or $b\overline{b}H$ alone. Thus, $N_{SD} = 2.25$ would be achieved for $b\overline{b}A$ or $b\overline{b}H$ along this contour were m_A and m_H widely separated.

We can then compute the statistical significance for the $b\overline{b}h_i$ and $b\overline{b}a_i$ signals with $h_i, a_i \to \tau\overline{\tau}$ decay using the following procedure. First, the NMSSM $b\overline{b}h_i$ and $b\overline{b}a_i$ production rates are related to the MSSM $b\overline{b}H$ and $b\overline{b}A$ rates by the factors $c_b(h_i)^2/\tan^2\beta$ and $c_b(a_i)^2/\tan^2\beta$, respectively. Next, we account for the fact that the $\tau\overline{\tau}$ branching ratios of the NMSSM scalars and pseudo-scalars differ somewhat from the value of 0.09 appropriate for the MSSM H and A. In particular, $BR(h_3 \to \tau\overline{\tau})$ is significantly reduced when the h_3 has large enough mass and large enough c_V that it acquires a modest WW^* branching ratio. Typical reductions will be tabulated in table 2. Thus, defining the value of $\tan\beta$ as a function of m_A shown by the $100\ \text{fb}^{-1}$ curve of fig. 19-62 in ref. [1] as $\tan\beta_{2.25}(m_A)$, we compute $N_{SD}(h_i)$ for $L = 100\ \text{fb}^{-1}$ as

$$N_{SD}(h_i) = 2.25 \left[\frac{c_b(h_i)}{\tan\beta_{2.25}(m_{h_i})} \right]^2 BR_{\tau\overline{\tau}}(h_i)\,, \tag{3.1}$$

where $BR_{\tau\overline{\tau}} \equiv BR(h_i \to \tau\overline{\tau})/BR(H \to \tau\overline{\tau}) = BR(h_i \to \tau\overline{\tau})/0.09$. An exactly parallel procedure is employed for $N_{SD}(a_i)$.

The above procedure is conservative in that it assumes no contribution to the $\tau\overline{\tau}$ channel N_{SD} from the gg fusion processes. However, as we shall describe in the next section, for the most difficult points in parameter space the gg-fusion rates are very substantially suppressed relative to MSSM values. For these points, essentially 99% of the $\tau\overline{\tau}$ channel N_{SD} would derive from $b\overline{b}$+Higgs associated production.

In recombining the scalar and pseudo-scalar signals, we must account for the fact that they can have fairly different masses in the NMSSM. In this paper, we have chosen to recombine the scalar and pseudo-scalar signals at different masses following the procedure of ref. [17], section 5.4, with $\sigma_m \sim 30$ GeV as estimated from fig. 19-61 in [1] at high luminosity and extrapolated to $m_A \lesssim 150$ GeV. This procedure leads to somewhat approximate estimates of the NMSSM statistical significances for this detection mode.

Using the above procedures, for each point in the parameter space of the NMSSM we obtain the statistical significances predicted for an integrated luminosity of $100\ \text{fb}^{-1}$ for each of the detection modes 1) – 9). In order to obtain the statistical significances for the various detection modes at 300 fb^{-1}, we multiply the 100 fb^{-1} statistical significances by $\sqrt{3}$ in the cases 1), 2), 3), 5) and 6), but only by a factor of 1.3 in the cases 4), 8) and 9). That such a factor is appropriate for mode 4), see, for example, fig. 19-62 in [1]. Use of this same factor for modes 8) and 9) is simply a conservative guess.

A detection mode that we do not explicitly consider in our analysis is that for the charged Higgs boson produced via $gb \to h^- t$ and $g\bar{b} \to h^+ \bar{t}$, following by $h^- \to \bar{t}b$ or $h^+ \to t\bar{b}$. This mode produces a 5σ signal with $L = 300$ fb^{-1} for $\tan\beta > 15$ or $\tan\beta < 4$ (see fig. 19-81 in ref. [1]) if $m_{h^+} > m_t + m_b$. In fact, as we shall see in the next section, the points for which LHC discovery of the neutral Higgs bosons is most difficult turn out to have $m_{h^+} \in [150 \text{ GeV}, 160 \text{ GeV}]$ which is precisely that region for which the mass is too large for observation in top decays and yet too small for on-shell $h^\pm \to tb$ decay.

4 Difficult Points

As stated in the introduction we still find "black spots" in the parameter space of the NMSSM, where the expected statistical significances for all Higgs detection modes 1) – 7) are below 5σ at 300 fb^{-1} (and even below 3σ in the worst cases). The reasons for this phenomenon have been described above; see also the corresponding discussion in [3]. However, after including the modes 8) and 9), the points that provide the worst 1) – 6) statistical significances typically yield robust signals in one or the other of the WW-fusion modes 8) and 9). When we scan for parameter choices for which the best statistical single-mode significances in 1) – 7) and, separately, 8) – 9) are simultaneously < 5, we find essentially only one small region of parameter space. A sample point from this very small region will be tabulated as point #1. At this point, the best single-channel significance for processes 1) – 6) and the best significance for processes 8) and 9) are both ~ 4.8. Thus, these two best signals are really both very robust. However, since they occur for different Higgs bosons, their statistical significances cannot be combined. Nonetheless, it is very unlikely that both signals would be missed.

In order to render the corresponding suppression mechanisms of the detection modes reproducible, we present the detailed properties of the above point and several other points in the parameter space in table 1. The notation is as follows: The bare parameters are as in eq. (2.5), with $m_{H_1}^2$, $m_{H_2}^2$ and m_S^2 fixed implicitly by the minimization conditions. (As noted earlier, with the convention λ, $\kappa > 0$ in the NMSSM, the sign of $\tan\beta$ can no longer be defined to be positive.) For the reasons discussed below eq. (2.4) we chose, in the stop sector, $X_t = \sqrt{6}$ for all of the points (1 – 6). For both scalar and pseudoscalar Higgs bosons, "gg Production Rate" denotes the ratio of the gluon-gluon production rate with respect to that obtained if $c_t = c_b = 1$, keeping the Higgs mass fixed. For scalar h_i, this is the same as the ratio of the gg production rate relative to that predicted for a SM Higgs boson of the same mass. For the scalar h_i, $BR\gamma\gamma$ denotes the ratio of the $\gamma\gamma$ branching ratio with respect to that of a SM Higgs boson with the same mass. (A verification of the reduced gluon-gluon production rates or $\gamma\gamma$ branching ratios would sometimes require the knowledge of the couplings to higher precision than given, for convenience, in table 1.) Also given for the scalar h_i are the ratios $BRb\overline{b}$ and $BRWW^*$ of the $b\overline{b}$ and WW^* branching ratios relative to the SM prediction. At tree-level (as noted above, we are neglecting loop corrections to the Yukawa couplings) one has $BR\tau\overline{\tau} = BRb\overline{b}$.

In table 2, we tabulate the statistical significances for the h_i in all the channels 1) – 9); production of the CP-odd a_i turns out to be relevant only when they add to the h_i signals in process 4). Also note that, even though we did not impose the requirement as part of the scan, all these problematical points are such that $m_{h_1} + m_{a_1} > 210$ GeV, so that $e^+e^- \to h_1 + a_1$ would have been kinematically forbidden at the highest LEP2 energy. Also tabulated in table 2 are four statistical significances obtained by combining various channels. This combination is done in the Gaussian approximation:

$$N_{SD}^{\text{combined}} = \left[\sum_i \left(N_{SD}^i \right)^2 \right]^{1/2} ,$$

where $\sum_i$ runs over the channels i being combined. We give results for the following combinations:

a) N_{SD} obtained by combining LHC channels 1) – 6);

b) N_{SD} obtained by combining LHC channels 1) – 6) and LEP2;

c) N_{SD} obtained by combining LHC channels 1) – 6) with the WW-fusion channels 8) and 9);

d) N_{SD} obtained by combining all LHC channels and LEP2, *i.e.* by combining all channels 1) – 9).

In those cases where there is no LEP2 signal, a)=b) and c)=d).

As summarized in table 3, all of the tabulated "bench mark points" have statistical significances below 5σ for all of the detection modes 1) – 6) at 300 fb^{-1} and 7) at LEP2. In more detail, as tabulated in table 2 and summarized in table 3, the best signals in the modes 1) – 6) for the points #1 – #6 at the LHC are:

- point #1, N_{SD} =4.85 for mode 5) and h_3;
- point #2, N_{SD} =3.22 for mode 5) and h_3;

- point #3, N_{SD} =3.73 for mode 2) and h_1;
- point #4, N_{SD} =3.44 for mode 3) and h_1;
- point #5, N_{SD} =2.82 for mode 4) and h_3;
- point #6, N_{SD} =4.54 for mode 4) and h_2;

Further, for many of these points, the combined statistical significance of modes 1) – 6) (also tabulated in table 3) would still be below 5 for any one h_i, although in all cases $\sqrt{2}N_{SD}^{1-6} > 5$ (as is likely to be relevant by combining ATLAS and CMS data once each detector has accumulated $L = 300$ fb^{-1}) for at least one of the h_i. However, for these "difficult" points the WW-fusion modes 8) and/or 9) provide (according to theoretical estimates) a decent (sometimes very strong) signal.

Only in the case of point #1 do the best WW-fusion $N_{SD}(8)$ and $N_{SD}(9)$ values also fall below 5. For this very worst case point, we still obtain $N_{SD}(5) = 4.9$ for h_3 and $N_{SD}(8) = 4.8$ for h_1 and h_2, signals that in practice would probably be readily seen. In addition, for this point $m_{h_1} = 104$ GeV and the ZZ coupling of the h_1 is sufficiently large that it would have yielded a 4.7σ signal at LEP2. In fact, the h_1 and h_2 are really almost degenerate ($m_{h_2} = 108$ GeV vs. $m_{h_1} = 104$ GeV) implying that their signals should be combined given the expected experimental resolution in the $\tau\overline{\tau}$ final state of mode 8) and that their overlap would bring the LEP2 mode 7) above the 5σ level. In short, the 4.9σ and 4.8σ signals predicted for h_1, h_2 and h_3 are rather robust and it is very unlikely that all three signals [or really two signals, that for mode 8) for the overlapping h_1/h_2 and that for mode 5) for the h_3] would remain unobserved at the LHC by virtue of simultaneous statistical fluctuations to the downside.

Point #2 is a case where all modes 1) – 6) yield significances below 3.3, and the signal in the WW-fusion mode 8) is not too pronounced [$N_{SD}(8) = 5.8$]. All points #2 – #6 require that the WW-fusion mode 8) is used in order to detect a Higgs boson. The points #2 – #5 differ as to which of the modes 1) – 6) and which h_i yields the largest statistical significance should the WW-fusion mode 8) not provide as strong a signal as suggested by the theoretical estimates. To render these points observable without the WW-fusion mode 8) would require improvements of all detection modes 2) – 5).

As in [3], we find that difficult points in the parameter space generally have $|\tan\beta| \sim$ 4–5. This is the region of $\tan\beta$ for which the $b\overline{b}h, b\overline{b}a$ signals are still not very much enhanced but yet the $gg \to h, a$ and $t\overline{t}h, t\overline{t}a$ signals have been suppressed somewhat. In a few cases, however, difficulties also arise for $|\tan\beta|$ as large as 8, as shown in the case of point #6. Also as in [3], the most difficult points are those in which the masses of the h_i and a_i are relatively close in magnitude, typically clustered in a ~ 60 GeV interval above ~ 105 GeV. Such clustering maximizes the mixing among the different Higgs bosons and thereby minimizes the significance of the discovery channels for any one Higgs boson. In particular, it is for strong mixing among the h_i that the statistical significance for discovery modes based on a large VV coupling for any one h_i are most easily suppressed.

5 Discussion and Conclusions

In this paper, we have addressed the question of whether or not it would be possible to fail to discover any of the Higgs bosons of the NMSSM using combined LEP2 and LHC data, possibly resulting in the erroneous conclusion that Higgs bosons with masses below 200 GeV have been excluded. We have demonstrated that, assuming that the decay channels i) to v) are either kinematically disallowed or render a Higgs boson observable, this is unlikely (at the $> 5\sigma$ level) to happen. Certainly, there are points in NMSSM parameter space for which the statistical significances for the individual detection modes 1) – 6) (*i.e.* those analyzed in detail by ATLAS and CMS) are all well below 5σ for integrated luminosity of 300 fb^{-1}. However, by combining several of the modes 1) – 6) and 300 fb^{-1} date from both ATLAS and CMS, a $> 5\sigma$ signal can be achieved based just on modes 1) – 6). Further, we have found that throughout all of the NMSSM parameter space (scanned subject to the earlier listed restrictions) for which such weak signals in modes 1) – 6) are predicted, the theoretical estimates for the WW-fusion modes indicate that an easily detected $WW \to h \to \tau\overline{\tau}$ signal should be present. Thus, our conclusion is that for all of the parameter space of the NMSSM compatible with reasonable boundary conditions for the parameters at the GUT scale (with, of course, non-universal soft terms in general) and such that Higgs pair and SUSY pair decays of the Higgs bosons are kinematically forbidden, at least one of the NMSSM Higgs bosons will be detected at the LHC. This is a big improvement over the results from the earlier Snowmass 1996 study which was somewhat negative without the inclusion of the $t\overline{t}h \to t\overline{t}b\overline{b}$ mode 4), and the WW-fusion modes 8) and 9).

It is amusing to note that all of our bench mark points for which Higgs discovery is most difficult at the LHC include one or two Higgs scalars with masses below ~ 117 GeV (with, however, reduced couplings to the Z boson), which could be responsible for the excess observed at LEP2 [18].

This study makes clear the importance of continuing to expand the sensitivity of existing modes and continuing to develop new modes for Higgs detection at the LHC in order not to have to wait for construction of a linear e^+e^- collider for detection of at least one of the SUSY Higgs bosons. In particular, study of the $a \to Zh$, $h \to aa$, $h \to h'h'$, $h, a \to t\overline{t}$ and SUSY pair channels should all be pushed. Of course, for the particular class of problematical points that we have emphasized, all the Higgs masses are below ~ 200 GeV so that $t\bar{t}$ decays will be kinematically highly suppressed (one of the top quarks would have to be virtual) and SUSY pair decays are quite unlikely to be significant given LEP2 limits on the masses of SUSY particles. Also, our difficult points tend to yield a charged Higgs boson that is only just beyond the reach of the $t \to H^+ b$ detection mode. Further improvement of this channel would be appropriate. Of course, by allowing Higgs masses such that the $a \to Zh$, $h \to aa$, $h \to h'h'$, and/or $h, a \to t\bar{t}$ decays are kinematically allowed we might find additional problematical points. We plan to pursue this issue in a future work.

Another important point that appears from our analysis is the fact that the full $L = 300$ fb^{-1} of integrated luminosity (per detector) is needed in order to have a robust "no-lose" Higgs discovery theorem for the NMSSM. Further, the fact that we find parameter space points for which the $L = 300$ fb^{-1} (per detector) signal for one of the Higgs bosons, even after combining all LHC modes [1) – 6), 8) and 9)] and data from both detectors, only reaches the 10σ level (c.f. sample points #1 and #2) suggests that SUSY models with more than one singlet might have parameter regions for which no Higgs could be discovered at the LHC. But, it is also possible that this is not the case. A detailed study, including overlapping signals would be required.

Of course, as in the MSSM, it is very possible that only one of the CP-even NMSSM Higgs bosons might be detected at the LHC but that, as studied by Kamoshita et al. in [12], the observation of all the CP-even Higgs bosons of the NMSSM would be possible at the LC by virtue of all having some non-negligible level of ZZ coupling and not having very high masses. Even at the LC, the CP-odd Higgs bosons might escape discovery, although this would not be the case for the parameter choices that we have found which make LHC discovery of even one NMSSM Higgs bosons most challenging. This is because, for such parameters, the a_i are relatively light and could be readily seen at the LC in the processes $e^+e^- \to h_i a_j$, $e^+e^- \to \nu\bar{\nu} a_i a_i$ and $e^+e^- \to Z^* \to Z a_i a_i$, assuming an integrated LC luminosity of 1000 fb^{-1} and energy $\sqrt{s} \geq 500$ GeV [19].

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Table Captions

Table 1: We tabulate the input bare model parameters, the corresponding Higgs masses, and the corresponding Higgs couplings, relative to SM Higgs boson coupling strength, for 6 bench mark points. Also given for the CP-even h_i are ratios of the gg production rate and various branching fractions relative to the values found for a SM Higgs of the same mass. For the CP-odd a_i, "gg Production Rate" refers to the value relative to what would be found if both the $b\overline{b}$ and the $t\overline{t}\ \gamma_5$ couplings had SM-like strength.

Table 2: Scalar Higgs statistical significances, $N_{SD} = S/\sqrt{B}$, in various channels for the 6 bench mark points. For each individual Higgs, we give (in order): N_{SD} for the channels 1) – 9) described in the text; Gaussian combined N_{SD} for non-WW-fusion LHC channels; combined N_{SD} for non-WW-fusion LHC channels plus LEP2; combined N_{SD} for all LHC channels, including the fusion channels $WW \to h \to \tau\overline{\tau}$ and $WW \to h \to WW^{(*)}$ channels; and combined N_{SD} for all LHC channels plus LEP2.

Table 3: Summary for all Higgs bosons. The entries are: maximum non-WW fusion LHC N_{SD}; maximum LHC WW fusion N_{SD}; best combined N_{SD} after summing over all non-WW-fusion LHC channels; and best combined N_{SD} after summing over all LHC channels. The Higgs boson for which these best values are achieved is indicated in the parenthesis. One should refer to the preceding table in order to find which channel(s) give the best values.

Table 1

Point Number	1	2	3	4	5	6
Bare Parameters						
λ	0.0205	0.0909	0.0325	0.0167	0.0325	0.1426
κ	0.0101	0.0679	0.0226	0.0075	0.0145	0.1549
$\tan\beta$	4.1	5.0	-5.0	5.0	5.0	-8.0
$\mu_{\rm eff}(GeV)$	130	100	-100	210	-150	-100
$A_\lambda(GeV)$	-36	-42	110	-80	40	130
$A_\kappa(GeV)$	-81	-129	90	-140	90	70
Scalar Masses and Couplings						
m_{h_1} (GeV)	104	105	110	109	106	115
c_V	0.60	0.64	0.63	0.81	0.50	0.85
c_t	0.51	0.59	-0.57	0.69	0.46	-0.79
c_b	2.12	1.74	1.98	3.75	1.51	4.76
gg Production Rate	0.21	0.30	0.47	0.35	0.18	1.06
$BR\gamma\gamma$	0.09	0.14	0.26	0.05	0.12	0.09
$BRb\overline{b} = BR\tau\overline{\tau}$	1.01	1.02	1.04	1.04	1.02	1.09
$BRWW^{(*)}$	0.08	0.14	0.10	0.05	0.11	0.03
m_{h_2} (GeV)	108	115	119	144	115	143
c_V	0.56	0.52	0.66	0.47	0.70	0.49
c_t	0.45	0.41	-0.59	0.60	0.61	-0.60
c_b	2.34	3.41	2.30	-3.03	3.08	-6.47
gg Production Rate	0.15	0.11	0.49	0.51	0.28	0.22
$BR\gamma\gamma$	0.07	0.03	0.22	0.06	0.06	0.04
$BRb\overline{b} = BR\tau\overline{\tau}$	1.03	1.09	1.14	2.83	1.09	2.74
$BRWW^{(*)}$	0.06	0.03	0.09	0.07	0.06	0.02
m_{h_3} (GeV)	150	150	158	150	152	202
c_V	0.57	0.56	0.42	0.37	0.51	0.20
c_t	0.77	0.72	-0.60	0.45	0.68	-0.19
c_b	-2.80	-3.36	-4.10	-1.67	-3.77	0.72
gg Production Rate	0.75	0.70	0.28	0.26	0.64	0.04
$BR\gamma\gamma$	0.14	0.10	0.28	0.17	0.08	0.00
$BRb\overline{b} = BR\tau\overline{\tau}$	3.98	4.16	11.9	4.04	5.17	12.7
$BRWW^{(*)}$	0.16	0.12	0.13	0.19	0.09	0.95
Pseudo-Scalar Masses and Couplings						
m_{a_1} (GeV)	124	133	135	134	132	111
c_t	0.06	0.19	0.06	0.20	0.11	0.10
c_b	1.04	4.83	1.60	5.00	2.75	6.13
gg Production Rate	0.00	0.02	0.00	0.02	0.01	0.04
m_{a_2}(GeV)	133	173	151	199	140	173
c_t	0.24	0.05	0.19	0.01	0.17	0.08
c_b	3.97	1.26	4.74	0.18	4.18	5.14
gg Production Rate	0.03	0.00	0.02	0.00	0.01	0.00
Charged Higgs Mass						
m_c(GeV)	155	157	170	156	159	159

Table 2

Point	1	2	3	4	5	6
Channel	h_1 Higgs boson					
$N_{SD}(1)$	0.34	0.81	2.47	0.37	0.39	2.02
$N_{SD}(2)$	1.22	2.13	3.73	0.96	1.37	1.69
$N_{SD}(3)$	2.10	2.71	2.34	3.44	1.63	3.98
$N_{SD}(4)$	1.49	2.01	0.43	2.72	0.71	4.30
$N_{SD}(5)$	0.00	0.00	0.00	0.00	0.00	0.18
$N_{SD}(6)$	0.08	0.21	0.28	0.09	0.10	0.24
$N_{SD}(7)$	4.71	3.98	0.57	1.66	2.39	0.00
$N_{SD}(8)$	4.84	5.77	6.58	10.7	3.60	14.8
$N_{SD}(9)$	0.00	0.00	0.29	0.20	0.00	0.31
$\sqrt{\sum_{i=1}^{6}[N_{SD}(i)]^2}$	2.87	4.07	5.08	4.50	2.28	6.31
$\sqrt{\sum_{i=1}^{7}[N_{SD}(i)]^2}$	5.51	5.69	5.11	4.80	3.30	6.31
$\sqrt{\sum_{i=1-6,8,9}[N_{SD}(i)]^2}$	5.62	7.07	8.32	11.6	4.26	16.1
$\sqrt{\sum_{i=1}^{9}[N_{SD}(i)]^2}$	7.33	8.11	8.33	11.8	4.88	16.1
Channel	h_2 Higgs boson					
$N_{SD}(1)$	0.20	0.07	2.61	0.49	0.39	0.15
$N_{SD}(2)$	0.81	0.34	3.30	0.50	0.97	0.35
$N_{SD}(3)$	1.52	1.06	2.09	0.91	2.32	0.96
$N_{SD}(4)$	1.57	2.59	0.57	2.67	2.37	4.54
$N_{SD}(5)$	0.00	0.01	0.33	1.14	0.08	0.11
$N_{SD}(6)$	0.05	0.02	0.36	0.77	0.11	0.08
$N_{SD}(7)$	1.73	0.00	0.00	0.00	0.00	0.00
$N_{SD}(8)$	4.84	5.72	10.5	6.85	10.4	7.89
$N_{SD}(9)$	0.07	0.09	0.78	0.90	0.37	0.24
$\sqrt{\sum_{i=1}^{6}[N_{SD}(i)]^2}$	2.34	2.82	4.76	3.22	3.48	4.66
$\sqrt{\sum_{i=1}^{7}[N_{SD}(i)]^2}$	2.91	2.82	4.76	3.22	3.48	4.66
$\sqrt{\sum_{i=1-6,8,9}[N_{SD}(i)]^2}$	5.38	6.37	11.6	7.62	10.9	9.16
$\sqrt{\sum_{i=1}^{9}[N_{SD}(i)]^2}$	5.65	6.37	11.6	7.62	10.9	9.16
Channel	h_3 Higgs boson					
$N_{SD}(1)$	1.44	0.99	0.81	0.61	0.65	0.00
$N_{SD}(2)$	1.24	0.88	1.13	0.92	0.56	0.00
$N_{SD}(3)$	0.00	0.00	0.00	0.00	0.00	0.00
$N_{SD}(4)$	1.77	2.62	3.00	2.17	2.82	1.86
$N_{SD}(5)$	4.85	3.22	1.63	1.96	2.47	2.57
$N_{SD}(6)$	3.81	2.53	1.67	1.56	2.07	0.65
$N_{SD}(7)$	0.00	0.00	0.00	0.00	0.00	0.00
$N_{SD}(8)$	0.00	0.00	0.00	0.00	0.00	0.00
$N_{SD}(9)$	3.06	2.16	1.86	1.52	1.53	0.00
$\sqrt{\sum_{i=1}^{6}[N_{SD}(i)]^2}$	6.69	5.04	4.05	3.50	4.37	3.24
$\sqrt{\sum_{i=1}^{7}[N_{SD}(i)]^2}$	6.69	5.04	4.05	3.50	4.37	3.24
$\sqrt{\sum_{i=1-6,8,9}[N_{SD}(i)]^2}$	7.35	5.48	4.45	3.81	4.63	3.24
$\sqrt{\sum_{i=1}^{9}[N_{SD}(i)]^2}$	7.35	5.48	4.45	3.81	4.63	3.24

Table 3

Point Number	1	2	3	4	5	6
Best non-WW fusion N_{SD}	4.85 (h_3)	3.22 (h_3)	3.73 (h_1)	3.44 (h_1)	2.82 (h_3)	4.54 (h_2)
Best WW fusion N_{SD}	4.84 ($h_{1,2}$)	5.77 (h_1)	10.5 (h_2)	10.7 (h_1)	10.4 (h_2)	14.8 (h_1)
Best combined N_{SD} w.o. WW-fusion modes	6.69 (h_3)	5.04 (h_3)	5.08 (h_1)	4.50 (h_1)	4.37 (h_3)	6.43 (h_1)
Best combined N_{SD} with WW-fusion modes	7.35 (h_3)	7.07 (h_1)	11.6 (h_2)	11.6 (h_1)	10.9 (h_2)	16.1 (h_1)

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